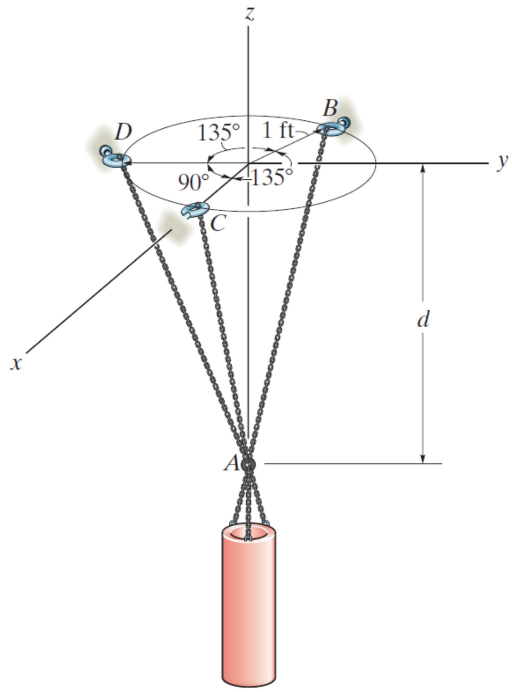
Problem 3-60

The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take $d = 1$ ft.



Prob. 3–60

Solution

Write position vectors to points A, B, C, and D.

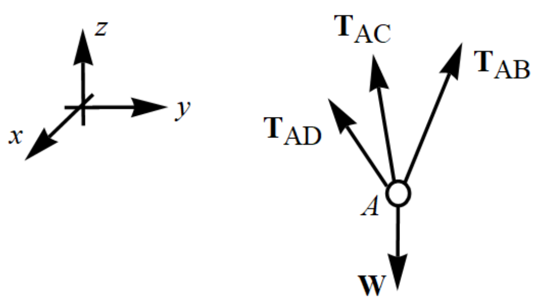
$$\mathbf{r}_A = \langle 0, 0, -d \rangle$$

$$\mathbf{r}_B = 1\langle \cos 135°, \cos(135° - 90°), 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = 1\langle 1, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_D = 1\langle 0, -1, 0 \rangle \text{ ft}$$

Draw a free-body diagram for the ring at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{W} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + W\langle 0, 0, -1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + W\langle 0, 0, -1\rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\langle \cos 135^\circ - 0, \cos 45^\circ - 0, 0 + d\rangle}{\sqrt{(\cos 135^\circ - 0)^2 + (\cos 45^\circ - 0)^2 + (0 + d)^2}} + T_{\text{AC}}\frac{\langle 1 - 0, 0 - 0, 0 + d\rangle}{\sqrt{(1 - 0)^2 + (0 - 0)^2 + (0 + d)^2}}$$
$$+ T_{\text{AD}}\frac{\langle 0 - 0, -1 - 0, 0 + d\rangle}{\sqrt{(0 - 0)^2 + (-1 - 0)^2 + (0 + d)^2}} + W\langle 0, 0, -1\rangle = \mathbf{0}$$

$$\frac{T_{\text{AB}}}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}\langle \cos 135^\circ, \cos 45^\circ, d\rangle + \frac{T_{\text{AC}}}{\sqrt{1 + d^2}}\langle 1, 0, d\rangle + \frac{T_{\text{AD}}}{\sqrt{1 + d^2}}\langle 0, -1, d\rangle + W\langle 0, 0, -1\rangle = \mathbf{0}$$

$$\Bigg\langle \frac{\cos 135^\circ}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}T_{\text{AB}} + \frac{1}{\sqrt{1 + d^2}}T_{\text{AC}},$$
$$\frac{\cos 45^\circ}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}T_{\text{AB}} - \frac{1}{\sqrt{1 + d^2}}T_{\text{AD}},$$
$$\frac{d}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}T_{\text{AB}} + \frac{d}{\sqrt{1 + d^2}}T_{\text{AC}} + \frac{d}{\sqrt{1 + d^2}}T_{\text{AD}} - W\Bigg\rangle = \langle 0, 0, 0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned} \frac{\cos 135^\circ}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}T_{\text{AB}} + \frac{1}{\sqrt{1 + d^2}}T_{\text{AC}} &= 0 \\ \frac{\cos 45^\circ}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}T_{\text{AB}} - \frac{1}{\sqrt{1 + d^2}}T_{\text{AD}} &= 0 \\ \frac{d}{\sqrt{\cos^2 135^\circ + \cos^2 45^\circ + d^2}}T_{\text{AB}} + \frac{d}{\sqrt{1 + d^2}}T_{\text{AC}} + \frac{d}{\sqrt{1 + d^2}}T_{\text{AD}} - W &= 0 \end{aligned}\right\}$$

Solving it and plugging in $W = 800$ lb and $d = 1$ ft yields

$$T_{\text{AB}} = \frac{(\sqrt{2} - 1)\sqrt{1 + d^2}}{d}W = 800(2 - \sqrt{2})\text{ lb} \approx 469\text{ lb}$$

$$T_{\text{AC}} = \frac{\sqrt{1 + d^2}}{(2 + \sqrt{2})d}W = 800(\sqrt{2} - 1)\text{ lb} \approx 331\text{ lb}$$

$$T_{\text{AD}} = \frac{\sqrt{1 + d^2}}{(2 + \sqrt{2})d}W = 800(\sqrt{2} - 1)\text{ lb} \approx 331\text{ lb.}$$